

CQE formula sheet (free, printable)

Descriptive statistics

Mean	$\bar{x} = \Sigma x / n$
Sample std dev	$s = \sqrt{[\Sigma (x - \bar{x})^2 / (n - 1)]}$
Variance of a sum (independent)	$\sigma^2 (X+Y) = \sigma^2 X + \sigma^2 Y$
Standard error of the mean	$\sigma \bar{x} = \sigma / \sqrt{n}$
z-score	$z = (x - \mu) / \sigma$
Coefficient of variation	$CV = s / \bar{x}$

Probability

Addition rule	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
Multiplication rule (independent)	$P(A \text{ and } B) = P(A) \times P(B)$
Binomial	$P(x) = C(n,x) p^x (1-p)^{n-x}$; mean np , variance $np(1-p)$
Poisson	$P(x) = e^{(-\lambda)} \lambda^x / x!$; mean = variance = λ
Combinations	$C(n,r) = n! / [r! (n-r)!]$

SPC: control limits

$\bar{X}$ -R chart	$\bar{X}: \bar{x} \pm A_2 \bar{R}$ R: $D_3 \bar{R}$ to $D_4 \bar{R}$
σ estimate from ranges	$\hat{\sigma} = \bar{R} / d_2$
Individuals-MR (n=2 moving range)	X: $\bar{x} \pm 2.66 \bar{MR}$ MR: UCL = $3.267 \bar{MR}$
p chart	$\bar{p} \pm 3\sqrt{[\bar{p}(1-\bar{p}) / n]}$
np chart	$n\bar{p} \pm 3\sqrt{[n\bar{p}(1-\bar{p})]}$
c chart	$\bar{c} \pm 3\sqrt{\bar{c}}$
u chart	$\bar{u} \pm 3\sqrt{(\bar{u} / n)}$
Rule	Negative lower limits are set to 0.

Control chart constants

n	A ₂	d ₂	D ₃	D ₄
2	1.880	1.128	0.000	3.267
3	1.023	1.693	0.000	2.574
4	0.729	2.059	0.000	2.282
5	0.577	2.326	0.000	2.114
6	0.483	2.534	0.000	2.004
7	0.419	2.704	0.076	1.924
8	0.373	2.847	0.136	1.864
9	0.337	2.970	0.184	1.816
10	0.308	3.078	0.223	1.777

Process capability

Cp / Pp	$(USL - LSL) / 6\sigma$ (σ within for Cp, σ overall for Pp)
Cpk / Ppk	$\min(USL - \bar{x}, \bar{x} - LSL) / 3\sigma$
Cpm (Taguchi)	$(USL - LSL) / 6\sqrt{[\sigma^2 + (\bar{x} - T)^2]}$
Defect rate from z	$z = (\text{limit} - \bar{x}) / \sigma \rightarrow$ read the tail from the normal table
Rule	Use only for a stable (in-control) process.

Measurement systems (MSA)

Total GRR	$\sigma^2_{\text{GRR}} = \sigma^2_{\text{repeatability}} + \sigma^2_{\text{reproducibility}}$
Total variation	$\sigma^2_{\text{total}} = \sigma^2_{\text{GRR}} + \sigma^2_{\text{part}}$
%GRR (study variation)	$100 \times \sigma_{\text{GRR}} / \sigma_{\text{total}}$
%GRR (tolerance)	$100 \times 6\sigma_{\text{GRR}} / (USL - LSL)$
Number of distinct categories	$ndc = 1.41 \times \sigma_{\text{part}} / \sigma_{\text{GRR}}$ (target ≥ 5)
Bias	bias = average measured – reference value

Acceptance sampling

Probability of acceptance (n, c)	$Pa = \sum \text{from } x=0 \text{ to } c \text{ of } C(n,x) p^x(1-p)^{n-x}$
AOQ (rectifying inspection, approx.)	$AOQ \approx Pa \times p$
Producer's risk α	$P(\text{reject} \mid \text{lot at AQL})$
Consumer's risk β	$P(\text{accept} \mid \text{lot at LTPD/RQL})$

Reliability

Failure rate (constant)	$\lambda = \text{failures} / \text{total operating time}$
MTBF / MTTF	$\theta = 1 / \lambda$
Exponential reliability	$R(t) = e^{(-\lambda t)}$ ($R(\theta) = 0.368$)
Series system	$R_{\text{sys}} = R_1 \times R_2 \times \dots \times R_n$
Parallel system	$R_{\text{sys}} = 1 - (1-R_1)(1-R_2)\dots(1-R_n)$
Availability	$A = \text{MTBF} / (\text{MTBF} + \text{MTTR})$
Weibull	$R(t) = e^{[-(t/\eta)^\beta]}$ $\beta < 1$ infant, $\beta \approx 1$ random, $\beta > 1$ wear-out

Design of experiments

Runs, full factorial	2^k (k factors, 2 levels)
Runs, fractional	$2^{(k-p)}$
Main effect	$\text{avg}(\text{response at } +) - \text{avg}(\text{response at } -)$
Coded coefficient	$\text{effect} / 2$
Resolution III / IV / V	main effects aliased with 2FI / 2FI aliased with each other / main effects and 2FI all clear

Hypothesis tests and intervals

1-sample z	$z = (\bar{x} - \mu_0) / (\sigma/\sqrt{n})$
1-sample t	$t = (\bar{x} - \mu_0) / (s/\sqrt{n}), df = n - 1$
2-sample t (Welch)	$t = (\bar{x}_1 - \bar{x}_2) / \sqrt{(s_1^2/n_1 + s_2^2/n_2)}$
Paired t	$t = \bar{d} / (s_d/\sqrt{n})$
Chi-square, one variance	$\chi^2 = (n-1)s^2 / \sigma_0^2$
F, two variances	$F = s_1^2 / s_2^2$ (larger on top)
1-proportion z	$z = (\hat{p} - p_0) / \sqrt{[p_0(1-p_0)/n]}$
Chi-square, counts	$\chi^2 = \sum (O - E)^2 / E$
CI for a mean	$\bar{x} \pm t(\alpha/2) \times s/\sqrt{n}$
Sample size for a mean	$n = [(z(\alpha/2) + z(\beta)) \sigma / \delta]^2$
Errors	$\alpha = P(\text{Type I}) = \text{reject true } H_0; \beta = P(\text{Type II}); \text{power} = 1 - \beta$

Regression and correlation

Slope	$b = \Sigma(x-\bar{x})(y-\bar{y}) / \Sigma(x-\bar{x})^2$
Intercept	$a = \bar{y} - b \bar{x}$
Coefficient of determination	$R^2 = 1 - SSE/SST$

Quality costs and metrics

DPMO	$\text{defects} \times 1,000,000 / (\text{units} \times \text{opportunities})$
First-pass yield	good first time / units in
Rolled throughput yield	$RTY = Y_1 \times Y_2 \times \dots \times Y_n$